

Question Paper

Quantitative Methods – I (131) : April 2004

1. If a straight line in **X-Y** plane has a negative slope, then [< Answer >](#)
- For a given value of **x**, the value of **y** will always be negative
 - It falls from left to right as the values increase along the **X**-axis
 - It always passes through the point of intersection of the **X** and **Y** axes
 - It is always parallel to the **Y**-axis
 - It is always parallel to the **X**-axis.
- (1 mark)
2. Which of the following is **false** with regard to the simplex method of solving a linear programming problem on profit maximization? [< Answer >](#)
- The pivot element is located at the intersection of pivot column and pivot row
 - The variable to leave solution can be identified by inspecting the values of $Z_j - C_j$ row
 - The simplex method can be applied when there are more than two decision variables
 - The values in the $Z_j - C_j$ row indicate whether the solution is optimal or not
 - The slack variables can assume non-negative values only.
- (1 mark)
3. Which of the following conditions indicates the existence of multiple optimal solutions when a linear programming problem is solved by the graphical method? [< Answer >](#)
- One of the constraints is parallel to the horizontal axis
 - The objective function is parallel to the vertical axis
 - The objective function is parallel to one of the edges of the feasible region which is in the direction of optimal movement of the objective function
 - All the decision variables assume negative values
 - One of the decision variables assumes negative values.
- (1 mark)
4. Which of the following statements is **true**? [< Answer >](#)
- The tallest rectangle in a histogram represents the modal class of the distribution
 - In a symmetrical distribution the mean, median and mode are unequal
 - The medians of two sets of data can be combined mathematically
 - The median can not be determined graphically
 - The mode is always uniquely defined.
- (1 mark)
5. The reciprocal of the harmonic mean is equal to [< Answer >](#)
- Arithmetic mean
 - Sum of all the observations
 - Sum of the reciprocals of the observations
 - Average of the reciprocals of the observations
 - Product of the reciprocals of the observations.
- (1 mark)
6. Which of the following is **false**? [< Answer >](#)
- If $b - a > 0$, then $a < b$
 - If $a < b$ and $b < c$, then $a < c$
 - If $a < b$, then $(a + c) < (b + c)$
 - If $a < b$ and $x < 0$, then $ax < bx$
 - If $a < b$ and $x = 0$, then $ax = bx$.
- (1 mark)
7. A quadratic equation is of the form $ax^2 + bx + c = 0$, where **a**, **b** and **c** are constants. If **c** is equal to zero, then the roots of the equation are [< Answer >](#)
- Only positive values
 - Only negative values

- (c) All equal to zero $\frac{b}{a}$ or $-\frac{b}{a}$ (e) 0 or $-\frac{b}{a}$.

(1 mark)

[< Answer >](#)

8. The events **B**, **C** and **D** are mutually exclusive and collectively exhaustive; **A** is another event which can jointly occur with **B**, **C** or **D**.

$$P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D) =$$

- (a) 1.00 (b) P(D) (c) P(C) (d) P(B) (e) P(A).

(1 mark)

[< Answer >](#)

9. In which of the following conditions two events, A and B, are said to be mutually exclusive?

- (a) $0 < P(A \text{ or } B) < 1$ (b) $P(A \text{ or } B) = 1$
 (c) $P(A) = P(B) = 1$ (d) $P(A/B) = 0$ and $P(B/A) = 0$
 (e) $0 < P(A \text{ and } B) < 1$.

(1 mark)

[< Answer >](#)

10. Which of the following intervals, defined by the end points **a** and **b**, excludes the real numbers which are less than or equal to **a**, or, greater than or equal to **b**?

- (a) (a, b) (b) [a, b]
 (c) (a, b] (d) [a, b]
 (e) None of the above.

(1 mark)

[< Answer >](#)

11. Which of the following measures **cannot** be combined mathematically?

- (a) Standard deviation (b) Arithmetic mean
 (c) Geometric mean (d) Harmonic mean (e) Median.

(1 mark)

[< Answer >](#)

12. According to Bienayme-Chebyshev theorem, at least what percentage of the observations in a distribution of data will lie within ± 2 standard deviations of the mean?

- (a) 25% (b) 30% (c) 45% (d) 60% (e) 75%.

(1 mark)

[< Answer >](#)

13. Which of the following indicates that the distribution curve of the data is negatively skewed?

- (a) Arithmetic Mean < Median < Mode
 (b) Median < Mode < Arithmetic Mean
 (c) Mode < Arithmetic Mean < Median
 (d) Mode < Median < Arithmetic Mean
 (e) Median < Arithmetic Mean < Mode.

(1 mark)

[< Answer >](#)

14. Which of the following graphical representations of data enables us to find out the number of observations which are less than certain values?

- (a) Less than ogive (b) More than ogive
 (c) Frequency polygon (d) Relative frequency polygon (e) Histogram.

(1 mark)

[< Answer >](#)

15. Which of the following is **true** with regard to a linear programming problem (LPP) on profit maximization?

- (a) Each structural constraint adds one column to the simplex tableau
 (b) In the simplex method inequations which contain ' $\leq$ ' are converted into equations by including slack variables
 (c) There can be only one feasible solution to a LPP
 (d) The slack variables make positive contributions to profit
 (e) None of the above.

(1 mark)

16. Which of the following is **not true** with regard to a logarithmic function $y = \log_b x$? [< Answer >](#)
- (a) The function is not defined for zero or negative values of x
 - (b) The value of the function is zero when $x = 1$
 - (c) The value of the function is negative when x lies between 0 and 1, and $b > 1$
 - (d) The value of the function decreases as the value of x increases for $b > 1$
 - (e) The value of the function is positive when $x > 1$.
- (1 mark)
17. The domain of a function contains [< Answer >](#)
- (a) Whole numbers only
 - (b) Natural numbers only
 - (c) The values that may be taken by the dependent variable
 - (d) The values that may be taken by the independent variable
 - (e) The values that may be taken by both the dependent and independent variables.
- (1 mark)
18. If the function $f(x)$ increases at a constant rate as x increases then [< Answer >](#)
- (a) $f'(x) = 0$
 - (b) $f'(x) < 0$ and $f''(x) > 0$
 - (c) $f'(x) < 0$ and $f''(x) = 0$
 - (d) $f'(x) > 0$ and $f''(x) < 0$
 - (e) $f'(x) > 0$ and $f''(x) = 0$.
- (1 mark)
19. Which of the following measures of central tendency may assume multiple values for the same set of data? [< Answer >](#)
- (a) Arithmetic mean
 - (b) Geometric mean
 - (c) Harmonic mean
 - (d) Median
 - (e) Mode.
- (1 mark)
20. Which of the following is **not true**? [< Answer >](#)
- (a) If the primal formulation of a linear programming problem is a maximizing problem then the dual formulation will be a minimizing problem
 - (b) If the primal formulation of a linear programming problem is a minimizing problem then the dual formulation will be a maximizing problem
 - (c) The dual formulation is an inverse of the primal formulation in every respect
 - (d) It is always easier to solve the primal formulation of a linear programming problem than the dual
 - (e) None of the above.
- (1 mark)
21. Which of the following is **true**? [< Answer >](#)
- (a) In probability theory, the result of an experiment is known as activity
 - (b) The probability of two or more statistically independent events occurring together is equal to the sum of their marginal probabilities
 - (c) Bayes' theorem is normally used to develop probabilities according to the classical approach
 - (d) The classical probability approach enables us to determine revised probabilities or posterior probabilities
 - (e) The set of all possible outcomes of an experiment is called the sample space of the experiment.
- (1 mark)
22. To which of the following distributions is Bienayme and Chebyshev's theorem applicable [< Answer >](#)
- (a) Normal Distribution
 - (b) Rectangular Distribution
 - (c) Symmetrical Distribution
 - (d) Non-symmetrical distribution
 - (e) Any type of distribution.
- (1 mark)
23. Which of the following measures represents the scatter of the values in a data set? [< Answer >](#)
- (a) Arithmetic mean
 - (b) Geometric mean
 - (c) Harmonic mean
 - (d) Median

(e) Standard deviation.

(1 mark)

24. Which of the following measures will remain unchanged when every observation in the data set is divided by the same quantity?

[< Answer >](#)

- (a) Range (b) Quartile deviation
(c) Standard deviation (d) Coefficient of variation (e) Mode.

(1 mark)

25. If every observation in a data set is increased by a constant quantity then the coefficient of variation of the resulting set of values will be

[< Answer >](#)

- (a) Less than the coefficient of variation of the original data set
(b) Greater than the coefficient of variation of the original data set
(c) Equal to the coefficient of variation of the original data set
(d) Equal to the coefficient of variation of the original data set plus the square root of the constant quantity
(e) Equal to the coefficient of variation of the original data set multiplied by the square root of the constant quantity.

(1 mark)

26. $\log_a b + \log_a c = 0$

[< Answer >](#)

This implies that

- (a) $b = c$ (b) $b = -c$
(c) $b + c = 1$ (d) $b - c = 1$
(e) b and c are reciprocals.

(1 mark)

27. Which of the following is **false** with regard to the derivative of a function?

[< Answer >](#)

- (a) It indicates the rate of change of the function at a given point
(b) The slope of the tangent to a function at a point is equal to the derivative of the function at the point
(c) The derivative may be a function of the independent variable
(d) The derivative of a linear function changes with the value of the independent variable
(e) If the derivative of a function at a point is negative then it indicates that the function is decreasing at that point.

(1 mark)

28. The least common multiple of a group of numbers is

[< Answer >](#)

- (a) The smallest number in the group
(b) A multiple of only the smallest number in the group
(c) A multiple of only the largest number in the group
(d) The smallest multiple of only the smallest number in the group
(e) The smallest number that can be divided by each number in a group of numbers without leaving a remainder.

(1 mark)

29. Which of the following statements is **true** with regard to the exponential function $y = m \cdot a^x$

[< Answer >](#)

- (a) The exponential curve falls from left to right as the values increase along the X-axis if $m > 0$ and $0 < a < 1$
(b) The exponential curve falls from left to right as the values increase along the X-axis if $m > 0$ and $a > 1$
(c) The exponential curve is parallel to the X-axis if $m > 0$ and $0 < a < 1$
(d) The exponential curve is parallel to the X-axis if $m > 0$ and $a > 1$
(e) None of the above.

(1 mark)

30. Baye's theorem helps the statistician to calculate

[< Answer >](#)

- (a) Subjective probability (b) Classical probability

- (c) Revised probability
(e) Dispersion.
- (d) Central tendency

(1 mark)

[< Answer >](#)

31. For a function, $y = f(x)$, $f'(x) > 0$ and $f''(x) < 0$. It can be said that $f(x)$ is
- (a) Constant for all values of x
(c) Decreasing at a decreasing rate
(e) Increasing at a decreasing rate.
- (b) Decreasing at an increasing rate
(d) Increasing at an increasing rate

(1 mark)

[< Answer >](#)

32. The domain of a function contains
- (a) The values which cannot be assumed by the independent variable
(b) The values which can be assumed by the independent variable
(c) The values which can be assumed by the dependent variable
(d) The values which cannot be assumed by the dependent variable
(e) Positive values only.

(1 mark)

[< Answer >](#)

33. Which of the following measures of dispersion is also called 'root mean square deviation'?
- (a) Range
(c) Mean absolute deviation
(e) None of the above.
- (b) Average deviation
(d) Standard deviation

(1 mark)

[< Answer >](#)

34. Which of the following is **true** with regard to the classical approach to probability?
- (a) Assumes that the outcomes are not equally likely
(b) The probability of an event is determined after performing the experiment large number of times
(c) The probability of an event is determined before performing the experiment
(d) It assumes that all possible outcomes of the experiment are not known
(e) The classical approach cannot be used to find out the probability of mutually exclusive events.

(1 mark)

[< Answer >](#)

35. Events **A** and **B** are dependent. The joint probability of the events **A** and **B** is
- (a) Equal to the product of the marginal probabilities of the events **A** and **B**
(b) Not equal to the product of the marginal probabilities of the events **A** and **B**
(c) Equal to the sum of the marginal probabilities of the events **A** and **B**
(d) Equal to the difference between the marginal probabilities of the events **A** and **B**
(e) Is always equal to 1.

(1 mark)

[< Answer >](#)

36. Which of the following is **not true** about interpolation and extrapolation?
- (a) Interpolation allows us to make insertions in a series of data
(b) Extrapolation allows us to forecast a value for some future date
(c) Both interpolation and extrapolation cannot be used when there are unexplained and violent fluctuations in the series of data
(d) Both interpolation and extrapolation assume that increases and decreases in the series of data take place at a uniform rate
(e) Both interpolation and extrapolation provide us with accurate values of the dependent variable.

(1 mark)

[< Answer >](#)

37. Which of the following is **true** about median?
- (a) Median is not useful as a measure of central tendency when the data set contains extreme values
(b) Median is not useful when the data set contains open ended classes
(c) Median can be calculated when the data set is not arranged in ascending or descending order
(d) Median is the middle item of a data set arranged in ascending or descending order
(e) Median is the value that has the highest frequency in a data set.

(1 mark)

[< Answer >](#)

38. Which of the following is **not** a property of the arithmetic mean?

- (a) The sum of the squared deviations of the items from the arithmetic mean is less than the sum of the squared deviations of the items from any other value
- (b) If all the items in the data set are reduced by the same quantity then the arithmetic mean of the resulting set of values is same as that of the original data set
- (c) If all the items in the data set are replaced by their arithmetic mean then their sum is equal to the sum of all the items in the original data set
- (d) The arithmetic means of two or more sets of data can be algebraically combined to find out the arithmetic mean of all the sets of data taken together
- (e) None of the above.

(1 mark)

[< Answer >](#)

39. Which of the following is **false** with regard to the graphical method of solving linear programming problems?

- (a) It is applicable when there are two decision variables
- (b) The decision variables are represented by the horizontal and vertical axes
- (c) Straight lines are used to demarcate the feasible region
- (d) The feasible region shows the solutions that satisfy all the constraints
- (e) One of the corner points of the feasible region will always be at the origin.

(1 mark)

[< Answer >](#)

40. In the graphical method of solving linear programming problems the feasible region is the set of all points

- (a) Which do not satisfy any of the constraints
- (b) Which satisfy exactly one of the constraints
- (c) Which satisfy all the constraints
- (d) At which the objective function has the same value
- (e) At which the objective function is equal to zero.

(1 mark)

[< Answer >](#)

41. If a, b, c are in H.P. and $(c + a)^2 > 4ac$, then the value of the expression $Z = \frac{a+b}{2a-b} + \frac{c+b}{2c-b}$ is
- (a) Less than 2
 - (b) Equal to 2
 - (c) More than 4
 - (d) Less than 3
 - (e) More than 5.

(2 marks)

[< Answer >](#)

42. If $n = 2000!$ then the summation of the series $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \dots + \frac{1}{\log_{2000} n}$ is equal to

- (a) 0
- (b) 1
- (c) $\log_n 2000$
- (d) $\log_2 2000!$
- (e) $\log_e 2001!$

(1 mark)

[< Answer >](#)

43. If $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, then find the value(s) of x .

- (a) 9, 4/13
- (b) 4, 9/13
- (c) 9/13, 4/13
- (d) 4/9, 9/13
- (e) 13.

(2 marks)

[< Answer >](#)

44. Evaluate the following limit $\lim_{x \rightarrow 3} \frac{(x-3)}{\sqrt{(x-2)} - \sqrt{(4-x)}}$

- (a) 0 (b) 1 (c) 2 (d) 3 (e) ∞ .

(1 mark)

[< Answer >](#)

45. A bag contains 30 black balls and 20 red balls. If 14 balls are drawn from the bag without replacement then the probability of drawing 8 black balls and 6 red balls is approximately

- (a) 0.125 (b) 0.242 (c) 0.625 (d) 0.75 (e) 0.81.

(1 mark)

[< Answer >](#)

46. If $y = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left[x + \sqrt{x^2 + a^2}\right]$, then the value of $\frac{dy}{dx}$ is equal to

- (a) $\frac{\sqrt{a^2 + x^2}}{\sqrt{a^2 - x^2}}$ (b) $\sqrt{a^2 - x^2}$ (c) $\frac{1}{\sqrt{a^2 + x^2}}$ (d) $\frac{1}{\sqrt{a^2 - x^2}}$ (e)

(2 marks)

[< Answer >](#)

47. How many different words can be formed with the letters of the word ALLAHABAD?

- (a) 120 (b) 7650 (c) 7560 (d) 40320 (e) 362880.

(1 mark)

[< Answer >](#)

48. A candidate is required to answer 7 out of 12 questions, which are divided in to two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. In how many different ways can we choose the 7 questions?

- (a) 180 (b) 210 (c) 600 (d) 780 (e) 792.

(1 mark)

[< Answer >](#)

49. Two dice are thrown together first and then three dice are thrown together. Find the probability that the total in the first throw is 4 or more and at the same time the total in the second throw is 6 or more.

- (a) $\frac{11}{12}$ (b) $\frac{11}{108}$ (c) $\frac{33}{112}$ (d) $\frac{103}{108}$ (e) $\frac{1133}{1296}$.

(2 marks)

[< Answer >](#)

50. If $(b + c)$, $(c + a)$ and $(a + b)$ are in H.P. then a^2 , b^2 and c^2 are in

- (a) A. P. (b) G. P. (c) H. P. (d) A.P. and G.P. both (e) An unknown progression.

(2 marks)

[< Answer >](#)

51. In a gamble Amit and Sumit each throw a pair of dice. Amit wins if he throws 6 before Sumit throws 7, and Sumit wins if he throws 7 before Amit throws 6. If Amit begins and the gamble continues indefinitely, then find out his chance of winning.

- (a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{5}{36}$ (d) $\frac{31}{36}$ (e) $\frac{30}{61}$.

(2 marks)

[< Answer >](#)

52. Find the sum of the following infinite series: $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots \infty$

- (a) $7/4$ (b) $3/4$ (c) $4/5$ (d) $11/8$ (e) $35/16$.

(2 marks)

[< Answer >](#)

53. Evaluate $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}$.

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 6.

(1 mark)

[< Answer >](#)

54. The sum of the first four terms of a geometrical progression (G.P.) is 60 and the sum of the first six terms of the same G.P. is 252. The common ratio is 2.

What is the tenth term of the G.P. from the beginning?

- (a) 64 (b) 256 (c) 1024 (d) 2048 (e) 4096.

(1 mark)

[< Answer >](#)

55. If p th, q th and r th terms of an A.P. are in G.P., then the common ratio of the G.P. is

- (a) $\frac{q+r}{p+q}$ (b) $\frac{q-r}{p-q}$ (c) $\frac{p-q}{q-r}$ (d) $\frac{p+q}{q+r}$ (e) $\frac{q-r}{p+q}$.

(2 marks)

[< Answer >](#)

56. In an A.P. if $S_m : S_n = m^2 : n^2$, where S_p denotes the sum of first p terms and t_p denotes the p th term, then the value of $t_m : t_n$ is equal to

- (a) $(2m+1) : (2n+1)$ (b) $(m+1) : (n+1)$
(c) $(2m-1) : (2n-1)$ (d) $(m-1) : (n-1)$ (e) $m : n$.

(2 marks)

[< Answer >](#)

57. If $S_1, S_2, S_3, \dots, S_n$ are the sums of infinite geometric series whose first terms are $1, 2, 3, \dots, n$ and whose common ratios are $1/2, 1/3, \dots, 1/(n+1)$.

Then $S_1 + S_2 + S_3 + \dots + S_n$ is equal to

- (a) $n(n+1)/2$ (b) $n(n+2)/2$
(c) $n(n+3)/2$ (d) $n(n+4)/2$ (e) $n(n+5)/2$.

(2 marks)

[< Answer >](#)

58. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$. Then the value of n is equal to

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

(2 marks)

[< Answer >](#)

59. $\frac{d}{dx} \left\{ \log_e \left(e^x \left(\frac{x-2}{x+2} \right)^{3/4} \right) \right\} =$

- (a) 1 (b) $e^x \left\{ \frac{x^2-1}{x^2-4} \right\}$ (c) $\frac{x^2-1}{x^2-4}$ (d) $\frac{x^2+1}{x^2-4}$ (e) e^x .

(1 mark)

[< Answer >](#)

60. If $z(x+y) = x^2 + y^2$, then $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 =$

(a) $\frac{(x-y)^2}{(x+y)^2}$ (b) $\frac{(x+y)^2}{(x-y)^2}$ (c) $\frac{2(x-y)^2}{(x+y)^2}$ (d) $\frac{3(x-y)^2}{(x+y)^2}$ (e) $\frac{4(x-y)^2}{(x+y)^2}$

(2 marks)

61. A vegetable trader foresees an increase in demand for onions over the next 3 months and wishes to enter the market at such a time that his profit from trading is maximized. Until the time he enters the market he shall only be procuring the onions from the growers. According to his assessment he can enter the market now with a supply of 10,000 kgs of onions and sell the entire lot at a profit of Rs.2.60 per kg. He further feels that if, instead of entering the market now, he waits for the market to pick up then he can increase his stock by 200 kgs of onions per day; however his profit will be reduced by 2 paise per kg per day.

The **number of days** the trader should wait before entering the market in order to maximize his profit is

- (a) 30 days (b) 35 days (c) 40 days (d) 45 days (e) 50 days.

(2 marks)

62. A test on mathematics has been conducted for the final year students of the graduation programme offered by The Hansraj College. The maximum marks for the test is 150 and 60 students appeared for the test. The marks obtained by the students in the test are given below:

Marks obtained by the students	Number of students
0 and above	60
25 and above	52
50 and above	42
75 and above	28
100 and above	16
125 and above	6

What is the **modal mark** obtained by students?

- (a) 46.54 (b) 54.46 (c) 66.67 (d) 70.54 (e) 72.50.

(2 marks)

63. The analysis of a day's production rejects resulted in the following figures:

No. of rejects per operator	21-25	26-30	31-35	36-40	41-45	46-50	51-55
No. of operators	5	15	28	42	15	12	3

The **standard deviation** of the number of rejects is

- (a) 4.245 (b) 5.275 (c) 6.735 (d) 7.325 (e) 8.125.

(2 marks)

64. A company has three establishments E_1 , E_2 , and E_3 in three cities. Analysis of the weekly salaries paid to the employees in the three establishments is given below:

	E_1	E_2	E_3
Number of employees	20	25	40
Average weekly salary (Rs)	305	300	340
Standard deviation of weekly salary (Rs)	50	40	45

The **combined standard deviation** of the weekly salaries of all 85 employees in the company is

- (a) Rs.44.56 (b) Rs.45.89 (c) Rs.46.76 (d) Rs.47.46 (e) Rs.48.69.

(3 marks)

[< Answer >](#)

65. Phoenix Instruments Ltd plans to produce two products Ammeters and Voltmeters to supply to college laboratories. These products undergo processing in two departments: Assembling and Testing. An Ammeter require 3 hours of processing time in assembling department and 2 hours of processing time in testing department and generates a profit of Rs100. A Voltmeter requires 2 hours of processing time in assembling department and 2 hours of processing time in testing department and generates a profit of Rs 75. The production hours available with assembling department and testing department are 4,800 and 3,600 respectively. The firm has adequate funds available with it for meeting the manufacturing costs of these instruments. The managers of Phoenix Instruments want to maximize their profit from these two products.

What is the **maximum profit** that Phoenix can generate with the optimal mix of the two products?

- (a) Rs. 135,000 (b) Rs. 160,000
(c) Rs. 165,000 (d) Rs. 170,000 (e) Rs. 175,000.

(3 marks)

[< Answer >](#)

66. Methodex Hardwares Ltd manufactures and sells a single type of pump in the local market. It has been experiencing a decline in its profitability. Its existing demand and cost functions are given as:

$$P = 70,000 - 3 Q$$

$$C = 0.0002 Q^2 - 5 Q + 81,000$$

Where, P = Price per unit (in Rs), C = Cost per unit (in Rs) and Q = Quantity sold in unit

The management of the firm has constituted a task force to undertake cost cutting measures. After reviewing the manufacturing process, the task force has suggested some modifications in the normal work practice that will reduce the cost by Rs. 10,000 per unit. The management has also decided to reduce the incentives to line managers that will reduce the cost burden by another Rs. 1,000 per unit. It is assumed that all the changes are implemented and the desired results are attained.

The **maximum profit** that can be attained by the sale of the pumps is

- (a) Rs. 1.86 Crores (b) Rs. 2.96 Crores
(c) Rs. 3.65 Crores (d) Rs. 4.76 Crores
(e) Rs. 5.26 Crores.

(3 marks)

[< Answer >](#)

67. A reputed newspaper company is planning to launch a magazine for teenagers in Maharashtra. The marketing team of the company has made a random survey of 200 college libraries in Maharashtra in order to find out the number of magazines they subscribe to. The survey report is given below:

Number of magazines	4-5	5-6	6-7	7-8	8-9
Number of college libraries	31	82	54	21	12

What is the **median** number of magazines subscribed by the college libraries?

- (a) 4.64 (b) 5.84 (c) 6.46 (d) 7.24 (e) 8.16.

(2 marks)

[< Answer >](#)

68. A pack of cards contains the following types of cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Each of these thirteen types of cards can be of four types – Spade, Club, Heart and Diamond. The pack of cards is shuffled well and four cards are drawn.

The **probability** that

- (a) Three out of the four cards are Aces is $48 / 270725$
(b) King, Queen, Jack and Ace are obtained in any order is $16 / 270725$
(c) Two cards are Kings and two cards are Queens is $192 / 270725$

- (d) Three cards are Aces and one card a King is 16 / 270725
 (e) At least one out of the four cards is an Ace is 145 / 270725.

(3 marks)

[< Answer >](#)

69. Gopal Agarwal, an investor has identified 30 stocks and 20 bonds. All the stocks have identical characteristics and all the bonds have identical characteristics. An experiment is conducted in which all the stocks and bonds are represented in a random order. Gopal is supposed to select exactly one security (i.e. a stock or a bond) randomly from the entire group of stocks and bonds. There is a 32% probability that a decision will be made for investment in the security selected by Gopal. There is a 24% probability that a stock is selected by Gopal and a decision to invest in it will be made.

The **probability** that

- (a) A bond is selected by Gopal and a decision to invest in it will be made is 24%
 (b) A bond is selected by Gopal and no decision to invest in it will be made is 28%
 (c) A stock is selected by Gopal and no decision to invest in it will be made is 36%
 (d) Gopal selected a bond if a decision for investment is made is 75%
 (e) Gopal selected a stock if a decision for investment is made is 25%.

(3 marks)

[< Answer >](#)

70. Find out the **sum** of the following series:

$$S = 4 \cdot \left(\frac{1}{2}\right) + 4 \cdot \left(\frac{1}{2}\right)^2 + 4 \cdot \left(\frac{1}{2}\right)^3 + 4 \cdot \left(\frac{1}{2}\right)^4 + \dots \infty$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

(1 mark)

[< Answer >](#)

71. Solve the following simultaneous equations and determine the value of **x**, **y** and **z** that satisfies all the three equations:

$$\begin{aligned} 2x - y + 3z &= 7 \\ x + y - 2z &= 6 \\ x + 2y - 4z &= 7 \end{aligned}$$

- (a) $x = 2, y = 5, z = -3$
 (b) $x = 3, y = 2, z = -5$
 (c) $x = -3, y = 2, z = 5$
 (d) $x = 5, y = -3, z = 2$
 (e) $x = 5, y = -3, z = -2$.

(1 mark)

[< Answer >](#)

72. There are two bags, one of which contains three black and four white balls while the other contains four black and three white balls. A die is cast: if the face 1 or 3 turns up, a ball is taken from the first bag; and if other face turns up, a ball is chosen from the second bag.

What is the probability of choosing a black ball?

- (a) 3/7 (b) 4/7 (c) 11/21 (d) 1/3 (e) 2/5.

(2 marks)

[< Answer >](#)

73. $\int_1^2 \frac{4x^3}{x^4 + 4} dx =$

(a) $\ln 2$ (b) $\ln 3$ (c) $\ln 4$ (d) $\ln 5$ (e) $\ln 6$.

(1 mark)

END OF QUESTION PAPER

Suggested Answers

Quantitative Methods – I (131) : April 2004

1. Answer : (b) [< TOP](#)
[>](#)
Reason : (a) A straight line with a negative slope does not necessarily mean that the value of y will always be negative for any given value of x.
(b) If a straight line has a negative slope then it falls from left to right as the values increase along the X-axis because for increase in the x – values will be accompanied with decrease in the y-values.
(c) A straight line passes through the origin when the y intercept is zero; a negative slope will not pass through the origin if the y intercept is not zero.
(d) A straight line is parallel to the Y-axis when the slope is infinite.
(e) A straight line is parallel to the X-axis when the slope is zero.

2. Answer : (b) [< TOP](#)
[>](#)
Reason : (a) The pivot element is located at the intersection of the pivot column and pivot row.
(b) The variable to enter solution can be identified by inspecting the values of the $Z_j - C_j$ row.
(c) The simplex method can be applied when there are more than two decision variables.
(d) The values in the $Z_j - C_j$ indicate whether the solution is optimal or not.
(e) The slack variables can assume non-negative values only.

3. Answer : (c) [< TOP](#)
[>](#)
Reason : (a) and (b) are not the indicators of multioptimality.
(c) In the graphical method of solving a LPP the situation of multioptimality arises when the objective function is parallel to one of the edges of the feasible region which is in the direction of the optimal movement of the objective function.
(d) and (e) are not correct because they only assume non-negative values.

4. Answer : (a) [< TOP](#)
[>](#)
Reason : (a) The tallest rectangle in a histogram represents the modal class.
(b) In a symmetrical distribution mean, median and mode are equal.
(c) The median can not be mathematically manipulated; hence medians of two sets of data can not be combined.
(d) The median can be determined graphically.
(e) The mode is not uniquely determined when more than one observations have the highest frequency.

5. Answer : (d) [< TOP](#)
[>](#)
Reason : Harmonic mean = Reciprocal of the average of reciprocals of the observations.
(a) The reciprocal of the harmonic mean is not equal to the arithmetic mean.
(b) The reciprocal of the harmonic mean is not equal to the sum of all the observations.
(c) The reciprocal of the harmonic mean is not equal to the sum of the reciprocals of the observations.
(d) The reciprocal of the harmonic mean is equal to average of the reciprocals of the observations.
(e) The reciprocal of the harmonic mean is not equal to the product of the reciprocals of the observations.

6. Answer : (d) [< TOP](#)
[>](#)
Reason : If $a < b$ and $x < 0$ then $ax > bx$. All other statements are true.

7. Answer : (e) [< TOP](#)
[>](#)
Reason : $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore \text{ For } c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 0}}{2a} = \frac{-b \pm b}{2a} = 0 \text{ or } -\frac{b}{a}$$

8. Answer : (e) [< TOP](#)
Reason : If B, C and D are mutually exclusive and collectively exhaustive, and A is another event which can jointly occur with B, C or D, then
 $P(A) = P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D)$.
This means that $P(A)$ is equal to the sum of all the joint probabilities which include event A. [>](#)
9. Answer : (d) [< TOP](#)
Reason : Since the two events A and B are mutually exclusive, the happening of A precludes the occurrence of B and vice versa.
Hence $P(A/B) = 0$ and $P(B/A) = 0$, [>](#)
10. Answer : (a) [< TOP](#)
Reason : (a, b) implies the set of all real numbers, x, such that, $a < x < b$. Therefore (a, b) excludes all real numbers $x \leq a$ and $x \geq b$.
[a, b] includes all real numbers, x, such that $a \leq x \leq b$.
(a, b] includes all real numbers, x, such that $a < x \leq b$
[a, b) includes all real numbers, x, such that $a \leq x < b$. [>](#)
11. Answer : (e) [< TOP](#)
Reason : (a) Standard deviations of two or more data sets can be mathematically combined
(b) Arithmetic means of two or more data sets can be mathematically combined
(c) Geometric means of two or more data sets can be mathematically combined
(d) Harmonic means of two or more data sets can be mathematically combined.
(e) Medians of two or more data sets cannot be mathematically combined [>](#)
12. Answer : (e) [< TOP](#)
Reason : According to Bienayme-Chebyshev theorem on the percentage of observations in a distribution
lying between $\pm 'k'$ standard deviations of the mean is $\left(1 - \frac{1}{k^2}\right) \times 100$
 $\therefore$ In this case it $\left(1 - \frac{1}{2^2}\right) \times 100 = 75\%$ [>](#)
13. Answer : (a) [< TOP](#)
Reason : In case of a negatively skewed distribution arithmetic mean < median < mode. Hence (a) is the answer. The relationships stated in (b), (c), (d) and (e) are not the features of negatively skewed distributions. [>](#)
14. Answer : (a) [< TOP](#)
Reason : A less than ogive enables us to find out the number of observations which are less than certain values. Hence (a) is the answer. All other graphical representations stated in (b), (c), (d) and (e) do not enable one to find out the number of observations which are less than certain values. [>](#)
15. Answer : (b) [< TOP](#)
Reason : In the simplex method of solving a LPP inequations containing ' $\leq$ ' are converted into equations by including slack variables. Each structural constraint adds one row in the simplex tableau. There can be many feasible solutions to a LPP. The slack variables make zero contributions to profit. Hence (b) is true and, (a), (c), (d) and (e) are not true. [>](#)
16. Answer : (d) [< TOP](#)
Reason : According to the properties of logarithmic functions all the alternatives (a) through (d) are true. Alternative (d) is not true because the value of the logarithmic function increases as the value of x increases, for $b > 1$. [>](#)
17. Answer : (d) [< TOP](#)

- Reason : Self explanatory. >
18. Answer : (e) [< TOP](#)
>
- Reason : If the function, $f(x)$, increases as x increases then, $f'(x)$ (the rate of change of 'x') is positive. If the function $f(x)$ increases at a constant rate then $f'(x)$ is a constant, hence $f''(x) = 0$.
19. Answer : (e) [< TOP](#)
>
- Reason : Mode can assume multiple values for the same data set if there are multiple values which occur the maximum number of times.
20. Answer : (d) [< TOP](#)
>
- Reason : There is no certainty that it is always easier to solve the primal formulation of a linear programming problem than the dual. All other alternatives are true with regard to the primal and dual formulations.
21. Answer : (e) [< TOP](#)
>
- Reason : The first four alternatives are clearly false. The fifth alternative is true. (Self explanatory.)
22. Answer : (e) [< TOP](#)
>
- Reason : Bienayme-Chebyshev theorem is applicable to any type of distribution regardless of its shape.
23. Answer : (e) [< TOP](#)
>
- Reason : The standard deviation represents the scatter of the values in a data set.
Arithmetic mean, geometric mean, harmonic mean and median are measures of central tendency.
24. Answer : (d) [< TOP](#)
>
- Reason : Coefficient of variation is a relative measure of dispersion. Range, quartile deviation, standard deviation and mode are absolute measures of dispersion. When every observation in the data set is divided by a constant both the standard deviation and the mean of the resulting data set will be divided by the constant. Since coefficient of variation is the ratio standard deviation to the mean, the coefficient of variation of the resulting data set will be same as the original data set. Since the other measures are absolute measures, they will permanently change due to the modification.
25. Answer : (a) [< TOP](#)
>
- Reason : If every observation in a data set is increased by a constant then the coefficient of variation of the resulting set of values will be less than the coefficient of variation of the original data set because the numerator remains the same whereas the denominator increases.
26. Answer : (e) [< TOP](#)
>
- Reason : (a) $b = c$ cannot be implied from the given condition.
(b) $b = -c$ cannot be implied from the given condition.
(c) $b + c = 1$ cannot be implied from the given condition.
(d) $b - c = 1$ cannot be implied from the given condition.
(e) Let $\log_a b = k$ and $\log_a c = m$.
Hence $b = a^k$ and $c = a^m$
 $\log_a b + \log_a c = 0$ implies that $k + m = 0$.
 $\therefore k = -m$
 $\Rightarrow a^k = a^{-m} \Rightarrow a^k = 1/a^m \Rightarrow a^k \cdot a^m = 1 \Rightarrow b \cdot c = 1 \Rightarrow b = 1/c$ and vice versa.
Hence they are reciprocals.
27. Answer : (d) [< TOP](#)
>
- Reason : (a) The derivative of a function indicates the rate of change of the function.
(b) (b) The slope of the tangent to a function at a point is equal to the derivative of the function at that point.
(c) The derivative of a function can be said to be a function of the independent variable if the expression of the derivative contains the independent variable.
(d) (d) The derivative of a linear function is the slope of the linear function, which is a constant value for all values of the dependent variable.

- (e) If the derivative of any function at a point is negative then it indicates that the function is decreasing at that point.

28. Answer : (e) [< TOP](#)
Reason : The least common multiple of a group of numbers is the smallest number that can be divided by each number of the group without leaving a remainder. [>](#)
29. Answer : (a) [< TOP](#)
Reason : The exponential function falls from left to right as the values increase along the X-axis if $m > 0$ and $0 < a < 1$. Hence (a) is true. (b), (c) (d) and (e) are not true because they are not the characteristics of the exponential function. [>](#)
30. Answer : (c) [< TOP](#)
Reason : Baye's theorem helps the statistician to calculate posterior (or revised) probability. [>](#)
31. Answer : (e) [< TOP](#)
Reason : If, for any function $y = f(x)$, $f'(x) > 0$ and $f''(x) < 0$ then the function is said to be increasing at a increasing rate. [>](#)
32. Answer : (b) [< TOP](#)
Reason : In a bi-variate function, the independent variable may assume any value within a certain interval. This interval of values is known as the domain of that function. [>](#)
33. Answer : (d) [< TOP](#)
Reason : Standard deviation is also called root mean square deviation. [>](#)
34. Answer : (c) [< TOP](#)
Reason : (a) The classical approach to probability assumes that the outcomes are equally likely.
(b) In the relative frequency approach to probability the probability of an event is determined after performing the experiment large number times.
(c) (c) In the classical approach to probability the probability of an event is determined before performing the experiment.
(d) The classical approach to probability assumes that all possible outcomes of the experiment are known.
(e) The classical approach can be used to find out the probability of mutually exclusive events. [>](#)
35. Answer : (b) [< TOP](#)
Reason : (a) & (b) For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the product of their marginal probabilities.
(c) For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the sum of their marginal probabilities.
(d) For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the difference between their marginal probabilities.
(e) For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not always equal to 1. [>](#)
36. Answer : (e) [< TOP](#)
Reason : Interpolation and extrapolation provide us only estimated values of the dependent variable. [>](#)
37. Answer : (d) [< TOP](#)
Reason : All other alternatives are false with regard to median except alternative (d). Refer to pages 71 and 72 of text book. [>](#)
38. Answer : (b) [< TOP](#)
Reason : All other alternatives are properties of the arithmetic mean. Refer to pages 69 and 70 of text book. [>](#)
39. Answer : (e) [< TOP](#)
Reason : The following are true with regard to the graphical method of solving LPPs:
a. It is applicable when there are two decision variables.
b. The decision variables are represented by the horizontal and vertical axes.
c. Straight lines are used to demarcate the feasible region. [>](#)

- d. The feasible region shows the solutions that satisfy all the constraints.
 e. The corner points of the feasible region may not include the origin.

40. Answer : (c)

Reason : The feasible region is the set off all points which satisfy all the constraints in the LPP.

[< TOP](#)
[>](#)

41. Answer : (c)

[< TOP](#)
[>](#)

Reason : As a, b, c are in H.P. $\therefore b = \frac{2ac}{a+c}$.

$$\frac{a+b}{2a-b} = \frac{a + \frac{2ac}{a+c}}{2a - \frac{2ac}{a+c}} = \frac{a^2 + 3ac}{2a^2} = \frac{1}{2} + \frac{3}{2} \cdot \frac{c}{a}$$

Now

$$\text{Similarly } \frac{c+b}{2c-b} = \frac{1}{2} + \frac{3}{2} \cdot \frac{a}{c}$$

Then the value of the expression

$$Z = \frac{1}{2} + \frac{3}{2} \cdot \frac{c}{a} + \frac{1}{2} + \frac{3}{2} \cdot \frac{a}{c} = 1 + \frac{3}{2} \left[\frac{c}{a} + \frac{a}{c} \right] = 1 + \frac{3}{2} \left[\frac{c^2 + a^2}{ac} \right] = 1 + 3 \left[\frac{c^2 + a^2}{2ac} \right]$$

But we know that $(c+a)^2 > 4ac$

$$\Rightarrow (c^2 + a^2) > 2ac$$

$$\Rightarrow \frac{c^2 + a^2}{2ac} > 1$$

$$\therefore Z > 1 + 3 \text{ or } Z > 4.$$

42. Answer : (b)

[< TOP](#)
[>](#)

Reason : Since $\log_b a = \frac{1}{\log_a b}$.

$$\begin{aligned} \text{Therefore the expression, } & \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \dots + \frac{1}{\log_{2000} n} \\ = & \log_n 2 + \log_n 3 + \log_n 4 + \dots + \log_n 2000 \\ = & \log_n (2 \cdot 3 \cdot 4 \dots 2000) \\ = & \log_n (2000!) = \log_n n = 1. \end{aligned}$$

43. Answer : (c)

[< TOP](#)
[>](#)

Reason : Given that $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$ or $\frac{x+1-x}{\sqrt{[x(1-x)]}} = \frac{13}{6}$

$$\text{or, } 169x(1-x) = 36$$

$$\text{or, } 169x^2 - 169x + 36 = 0$$

$$\text{or, } 169x^2 - 117x - 52x + 36 = 0$$

$$\text{or, } 13x(13x-9) - 4(13x-9) = 0$$

$$\text{or, } (13x-9)(13x-4) = 0$$

$$\therefore x = \frac{9}{13}, \frac{4}{13}$$

44. Answer : (b)

[< TOP](#)
[>](#)

Reason :
$$\lim_{x \rightarrow 3} \frac{(x-3)}{\sqrt{(x-2)} - \sqrt{(4-x)}}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)}{\sqrt{(x-2)} - \sqrt{(4-x)}} \times \frac{\sqrt{(x-2)} + \sqrt{(4-x)}}{\sqrt{(x-2)} + \sqrt{(4-x)}}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3) \times [\sqrt{(x-2)} + \sqrt{(4-x)}]}{(x-2) - (4-x)}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3) \times [\sqrt{(x-2)} + \sqrt{(4-x)}]}{2(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{\sqrt{(x-2)} + \sqrt{(4-x)}}{2} \text{ for } (x \neq 3)$$

$$= \frac{\sqrt{(3-2)} + \sqrt{(4-3)}}{2} = \frac{2}{2} = 1.$$

45. Answer : (b)

[< TOP](#)
[>](#)

Reason :
$$\begin{array}{rcl} \text{Total number of balls} & = & 30 + 20 = 50 \\ \text{Number of black balls} & = & 30 \\ \text{Number of red balls} & = & 20 \end{array}$$

Number of ways in which 8 black balls can be selected from 30 = ${}^{30}C_8$

Number of ways in which 6 red balls can be selected from 20 = ${}^{20}C_6$

Number of ways in which 8 black and 6 red balls can be selected = ${}^{30}C_8 \times {}^{20}C_6$

Number of ways in which 14 balls can be selected from 50 = ${}^{50}C_{14}$

$$\therefore P(8 \text{ black balls and } 6 \text{ red balls}) = \frac{{}^{30}C_8 \times {}^{20}C_6}{{}^{50}C_{14}} = 0.242 \text{ (approx)}$$

46. Answer : (a)

[< TOP](#)
[>](#)

Reason :
$$y = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log \left[x + \sqrt{x^2 + a^2} \right]$$

Therefore,

$$\frac{dy}{dx} = \left[\frac{1}{2} \sqrt{a^2 + x^2} + \frac{x}{2} \cdot \frac{1}{2\sqrt{a^2 + x^2}} \cdot 2x \right] + \frac{a^2}{2} \cdot \left[\left(\frac{1}{x + \sqrt{x^2 + a^2}} \right) \cdot \left(1 + \frac{2x}{2\sqrt{x^2 + a^2}} \right) \right]$$

$$= \frac{1}{2\sqrt{a^2 + x^2}} (a^2 + 2x^2) + \frac{a^2}{2} \frac{1}{\sqrt{a^2 + x^2}}$$

$$= \frac{(a^2 + 2x^2 + a^2)}{2\sqrt{a^2 + x^2}} = \sqrt{a^2 + x^2}$$

47. Answer : (c)

[< TOP](#)
[>](#)

Reason : If out of n things p are exactly alike of one kind and q exactly alike of second kind and r exactly alike of third kind and the rest all different then the number of permutations of n things taken all

$$\text{at a time} = \frac{n!}{p! \cdot q! \cdot r!}$$

In the word 'ALLAHABAD' there are 4 A, 2 L and 1 each of H, B and D. Total number of letters are 9.

Therefore the number of words that can be formed with these letters = $\frac{9!}{4! \cdot 2!} = 7560$.

48. Answer : (d)

[< TOP](#)
[>](#)

Reason : The number of ways the candidate can choose 7 questions is

(5A, 2B), (4A, 3B), (3A, 4B) and (2A, 5B), where 5A, 2B means 5 questions from part A and 2 question from part B.

$${}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_2 \times {}^6C_5 \\ = 90 + 300 + 300 + 90 = 780$$

49. Answer : (e)

[< TOP](#)
[>](#)

Reason : In the first throw a number less than 4 can come as (1,1), (1,2), (2,1) therefore total in the first throw could be 4 or more in $36 - 3 = 33$ ways. Since total number of ways the two dice can be thrown is 36 ways.

$$\text{Hence probability in this case} = P(A) = \frac{33}{36} = \frac{11}{12}$$

In the second case, three dice are thrown. In this case, the total of 3, 4 or 5 can come as (1,1,1), (2,1,1), (1,2,1), (1,1,2), (1,1,3), (3,1,1), (1,3,1), (1,2,2), (2,1,2) and (2,2,1) i.e. in 10 ways.

Hence the number 6 or more can come in $216 - 10 = 206$ ways.

$$\text{Hence probability in this case} = P(B) = \frac{206}{216} = \frac{103}{108}$$

Therefore the required probability,

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{11}{12} \times \frac{103}{108} = \frac{1133}{1296}$$

50. Answer : (a)

[< TOP](#)
[>](#)

Reason : Taking the reciprocals $\frac{1}{b+c}$, $\frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

$$\therefore \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\text{or, } \frac{b-a}{(c+a)(b+c)} = \frac{c-b}{(a+b)(c+a)}$$

Canceling $(c+a)$ and cross multiplying, we get,

$$b^2 - a^2 = c^2 - b^2$$

$$\therefore a^2, b^2 \text{ and } c^2 \text{ are in A.P.}$$

51. Answer : (e)

[< TOP](#)
[>](#)

Reason : In a single throw with a pair of dice, the total number of ways is 36. The number of ways of throwing 6 is 5, and the number of ways of throwing 7 is 6. Hence if p_1 and p_2 denote the probabilities of throwing 6 and 7 respectively in a single throw with a pair of dice, then

$$p_1 = \frac{5}{36} \text{ and } p_2 = \frac{6}{36} = \frac{1}{6}$$

$$\text{Therefore } q_1 = \bar{p}_1 = 1 - \frac{5}{36} = \frac{31}{36} \text{ and } q_2 = \bar{p}_2 = 1 - \frac{1}{6} = \frac{5}{6}$$

Now Amit will win if he throws 6 at the first throw or at the third throw when Amit fails to throw 6 at the first and Sumit fails to throw 7 at the second throw or at the 5th throw when Amit fails twice to throw 6 and Sumit fails twice to throw 7 and so on.

Hence Amit's chance of winning

$$= p_1 + q_1 \cdot q_2 \cdot p_1 + q_1^2 \cdot q_2^2 \cdot p_1 + \dots \infty$$

$$= \frac{5}{36} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} + \left(\frac{31}{36}\right)^2 \cdot \left(\frac{5}{6}\right)^2 \cdot \frac{5}{36} + \dots \infty$$

$$= \frac{\frac{5}{36}}{1 - \frac{31}{36} \cdot \frac{5}{6}} = \frac{5}{36} \times \frac{216}{61} = \frac{30}{61}$$

52. Answer : (e)

[<TOP](#)
[>](#)

Reason : Let $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots \infty$

Multiplying the above series with $(1/5)$, we have,

$$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \dots \infty$$

$$\text{Therefore } \left(1 - \frac{1}{5}\right)S = 1 + 3\left[\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \dots \infty\right]$$

$$\text{Or } \frac{4}{5}S = 1 + 3 \cdot \frac{1/5}{1 - (1/5)} = 1 + \frac{3}{4} = \frac{7}{4}$$

$$\therefore S = \frac{35}{16}$$

53. Answer : (c)

[<TOP](#)
[>](#)

Reason : Let $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}$

It can be rewritten as, $y = \sqrt{6 + y}$

Squaring both sides, $y^2 = 6 + y$

$$\text{or, } y^2 - y - 6 = 0$$

$$\text{or, } (y - 3)(y + 2) = 0$$

The value of $y = 3$ and -2 . The negative value is rejected. Therefore the value of the expression is 3.

54. Answer : (d)

[<TOP](#)
[>](#)

Reason : Sum of the first four terms, $S_4 = 60$

Sum of the first six terms, $S_6 = 252$

$$S_6 - S_4 = 252 - 60$$

or $S_6 - S_5 + S_5 - S_4 = 192$ (where S_5 is the sum of first five terms)

$$\text{or } (S_6 - S_5) + (S_5 - S_4) = 192 \quad \dots \quad (A)$$

Now, $S_6 = S_5 + t_6$ (where t_6 is the sixth term)

$$\Rightarrow S_6 - S_5 = t_6$$

$S_5 = S_4 + t_5$ (where t_5 is the fifth term)

$$\Rightarrow S_5 - S_4 = t_5$$

$\therefore$ Equation (A) may be rewritten as $t_6 + t_5 = 192$

Now, $t_6 = t_5 \times r$ (since the terms are in G.P. each term will be equal to the product of its preceding term and common ratio, r).

$$\therefore t_5 \cdot r + t_5 = 192$$

$$\text{or } t_5(r + 1) = 192$$

$$\text{or } t_5 = \frac{192}{r + 1}$$

$$\therefore t_5 = \frac{192}{2 + 1} = 64 \quad (r = 2, \text{ given})$$

$t_5 = ar^{5-1} = ar^4$ (where 'a' is the first term)

$$\therefore a(2^4) = 64 \quad \text{or} \quad a = \frac{64}{2^4} = 4$$

$$\therefore t_{10} = ar^{10-1} = ar^9 = 4(2^9) = 2048$$

55. Answer : (b)

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Reason : We are given that T_p, T_q and T_r of an A.P. are in G.P.

$$\therefore \frac{T_q}{T_p} = \frac{T_r}{T_q} = R$$

(common ratio of G.P.)

$$\frac{a + (q-1)d}{a + (p-1)d} = \frac{a + (r-1)d}{a + (q-1)d}$$

From the property of ratio we know that if $\frac{x}{y} = \frac{z}{w}$, then the ratio is equal to $\frac{x-z}{y-w}$ or $\frac{x+z}{y+w}$
Using the property we have,

$$R = \frac{[a + (q-1)d] - [a + (r-1)d]}{[a + (p-1)d] - [a + (q-1)d]} = \frac{(q-r)d}{(p-q)d} = \frac{q-r}{p-q}$$

56. Answer : (c)

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Reason : Let a be the first term and d be the common difference of the A.P.

$$\therefore S_m = \frac{m}{2}[2a + (m-1)d] \quad \text{and} \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_m : S_n = m^2 : n^2 \Rightarrow \frac{m}{n} \left[\frac{2a + (m-1)d}{2a + (n-1)d} \right] = \frac{m^2}{n^2} \Rightarrow 2a(n-m) = d(n-m) \Rightarrow 2a = d$$

$$\begin{aligned} \text{Now, } t_m : t_n &= [a + (m-1)d] : [a + (n-1)d] = (a + 2am - 2a) : (a + 2an - 2a) \\ &= (2m-1) : (2n-1) \end{aligned}$$

57. Answer : (c)

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Reason : For the G.P. with first term n and common ratio $1/(n+1)$

$$\frac{n}{1 - \frac{1}{n+1}} = n+1$$

$$\therefore S_1 + S_2 + S_3 + \dots + S_n = \sum S_i = \sum (i+1) = \sum i + n = \frac{n(n+1)}{2} + n = \frac{n(n+3)}{2}$$

58. Answer : (d)

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$$\text{Reason : We have } 5. \frac{(2n+1)!}{(n+2)!} = 3. \frac{(2n-1)!}{(n-1)!}$$

$$\Rightarrow 5. \frac{(2n+1)2n(2n-1)!}{(n+2)(n+1)n(n-1)!} = 3. \frac{(2n-1)!}{(n-1)!}$$

$$\Rightarrow 10(2n+1) = 3(n+2)(n+1)$$

$$\Rightarrow 20n+10 = 3n^2 + 9n+6$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\text{or, } (n-4)(3n+1) = 0$$

$$\text{Therefore, } n = 4.$$

59. Answer : (c)

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$$\text{Reason : } y = \log_e \left(e^x \left(\frac{x-2}{x+2} \right)^{3/4} \right)$$

$$\Rightarrow y = \log_e e^x + \frac{3}{4} [\log(x-2) - \log(x+2)]$$

$$\Rightarrow y = x + \frac{3}{4}[\log(x-2) - \log(x+2)]$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{3}{4}\left[\frac{1}{x-2} - \frac{1}{x+2}\right]$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{3}{4}\left[\frac{4}{x^2-4}\right]$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{3}{x^2-4} = \frac{x^2-1}{x^2-4}$$

60. Answer : (e)

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Reason : Given that $z = \frac{x^2 + y^2}{x + y}$

$$\therefore \frac{\partial z}{\partial x} = \frac{(x+y)2x - (x^2 + y^2).1}{(x+y)^2} = \frac{x^2 + 2xy - y^2}{(x+y)^2}$$

$$\text{and } \frac{\partial z}{\partial y} = \frac{(x+y)2y - (x^2 + y^2).1}{(x+y)^2} = \frac{y^2 + 2xy - x^2}{(x+y)^2}$$

$$\therefore \left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = \left[\frac{x^2 + 2xy - y^2}{(x+y)^2} - \frac{y^2 + 2xy - x^2}{(x+y)^2}\right]^2$$

$$= \left[\frac{2x^2 - 2y^2}{(x+y)^2}\right]^2 = \frac{4(x-y)^2}{(x+y)^2}$$

61. Answer : (c)

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Reason : Let the vegetable trader wait 'x' days before he enters the market.

$\therefore$ Quantity of onion that he can sell when he enters the market = $10,000 + 200x$

Profit per kg = $2.60 - 0.02x$

Let $f(x)$ denote the profit function (in Rs.)

$$\begin{aligned}\therefore f(x) &= (10,000 + 200x)(2.60 - 0.02x) \\ &= 26,000 - 200x + 520x - 4x^2\end{aligned}$$

$$\text{or } f(x) = 26,000 + 320x - 4x^2$$

Profit will be maximized at that value of 'x' for which

$$f'(x) = 0 \text{ and } f''(x) < 0$$

$$\begin{aligned}\therefore f'(x) &= 320 - 8x = 0 \\ \text{or } 320 &= 8x\end{aligned}$$

$$\text{or } x = \frac{320}{8} = 40 \text{ days.}$$

$$f''(x) = -8 < 0 \text{ for all values of } x.$$

$\therefore$ The profit is maximized if the vegetable trader waits for 40 days before entering the market.

62. Answer : (c)

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Reason : The cumulative frequency distribution (more than type) is given. The frequency distribution of the marks obtained is derived from the same and given below:

Marks	Cumulative frequency (cf)	Frequency (f)
-------	---------------------------	---------------

	(more than)	
0 – 25	60	60 – 52 = 8
25 – 50	52	52 – 42 = 10
50 – 75	42	42 – 28 = 14
75 – 100	28	28 – 16 = 12
100 – 125	16	16 – 6 = 10
125 – 150	6	6
		$\Sigma f = 60$

$$\text{Mode} = L_{mo} + \left[\frac{f - f_1}{2f - f_1 - f_2} \right] w$$

Modal class is the class with the highest frequency. In this case the class 50–75 has the highest frequency (14). So the Modal class is 50–75.

$$\therefore L_{mo} = 50, f = 14, f_1 = 10, f_2 = 12, w = 25$$

$$\therefore \text{Mode} = 50 + \left[\frac{14 - 10}{2 \times 14 - 10 - 12} \right] 25 = 50 + \left[\frac{4}{28 - 22} \right] 25 = 66.67$$

$$\therefore \text{Modal marks} = 66.67$$

63. Answer : (c)

Reason : Converting the discrete data into the continuous data, we get the following table:

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No. of rejects Per operator	M.P. (X)	No. of Operators (f)	(X-38)/5 (d)	fd	fd ²
20.5 - 25.5	23	5	-3	-15	45
25.5 - 30.5	28	15	-2	-30	60
30.5 - 35.5	33	28	-1	-28	28
35.5 - 40.5	38	42	0	0	0
40.5 - 45.5	43	15	1	15	15
45.5 - 50.5	48	12	2	24	48
50.5 - 55.5	53	3	3	9	27
Total		120		-25	223

$$\text{Standard deviation : } \sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N} \right)^2} \times i$$

$$= \sqrt{\frac{223}{120} - \left(\frac{-25}{120} \right)^2} \times 5$$

$$= \sqrt{1.858 - .043} \times 5 = 1.347 \times 5 = 6.735$$

64. Answer : (e)

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$$\text{Reason : } \bar{X}_{123} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2 + N_3 \bar{X}_3}{N_1 + N_2 + N_3} = \frac{20 \times 305 + 25 \times 300 + 40 \times 340}{20 + 25 + 40} = 320$$

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1d_1^2 + N_2d_2^2 + N_3d_3^2}{N_1 + N_2 + N_3}}$$

$$d_1 = |\bar{X}_1 - \bar{X}_{123}| = |305 - 320| = 15$$

$$d_2 = |\bar{X}_2 - \bar{X}_{123}| = |300 - 320| = 20$$

$$d_3 = |\bar{X}_3 - \bar{X}_{123}| = |340 - 320| = 20$$

$$\begin{aligned}\sigma_{123} &= \sqrt{\frac{20(50)^2 + 25(40)^2 + 40(45)^2 + 20(15)^2 + 25(20)^2 + 40(20)^2}{20 + 25 + 40}} \\ &= \sqrt{\frac{50000 + 40000 + 81000 + 4500 + 10000 + 16000}{85}} \\ &= \sqrt{\frac{201500}{85}} = 2370.59 = 48.69\end{aligned}$$

Thus the combined average weekly salary is Rs 320 and the combined standard deviation is Rs 48.69.

65. Answer : (c)

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Reason : The linear programming problem can be formulated as below,

Let x be the optimum number of Ammeters to be produced for maximizing the profit and y be the optimum number of Voltmeters to be produced to maximum profit. Then the profit obtained is

$Z = 100x + 75y$, where Z is the profit generated through the sale of Ammeters and Voltmeters.

Each unit of Ammeter requires 3 hours of assembling time while each voltmeter requires 2 hours of assembling time. Total number of assembling hours is limited to 4,800 hours. Therefore,

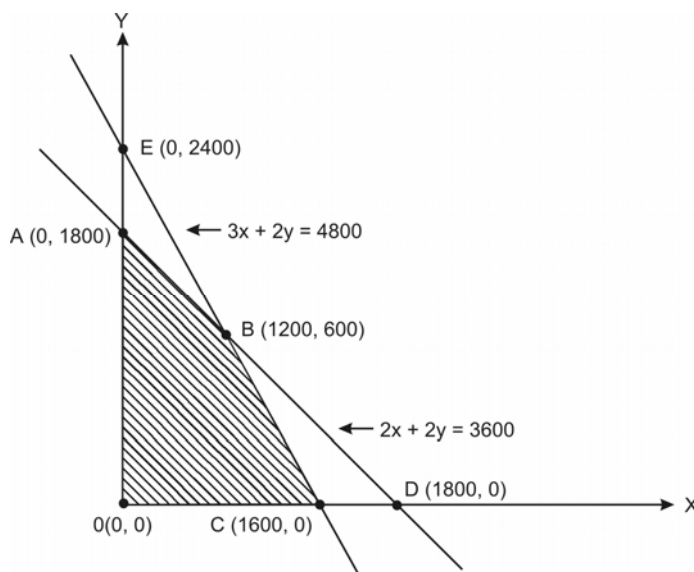
$$3x + 2y \leq 4,800.$$

Each unit of Ammeter requires 2 hours of testing time and each Voltmeter requires 2 hours of testing time. The total number of hours available for testing is 3,600 hours. Therefore,

$$2x + 2y \leq 3,600.$$

The number of Ammeters and Voltmeter to be produced for maximum profit cannot take any negative values. Therefore,

$$x \geq 0 \text{ and } y \geq 0.$$



The assembling constraint is depicted by line EC and the testing constraint is shown by line AD. The Feasible region is shown by OABC.

Now we know that the optimum point will occur at one of the corner points. So calculating the profit at the corner points: $Z = 100x + 75y$

Point	Coordinates	Profit (Z) in Rs.
O	(0,0)	0
A	(0, 1800)	135,000
C	(1600, 0)	160,000
B	(1200, 600)	165,000

We see that the maximum profit occurs at the point B. Therefore a maximum profit of **Rs 165,000** is achieved by producing **1200** units of Ammeters and **600** units of Voltmeters.

66. Answer : (b)

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Reason : Given that $P = 70,000 - 3Q$

$$\text{And } C = 0.0002Q^2 - 5Q + 81,000.$$

But according to the question, application of cost cutting measures has further reduced the cost of each unit. Therefore the new cost function is,

$$\begin{aligned} C &= 0.0002Q^2 - 5Q + 81,000 - (10,000 + 1,000) \\ &= 0.0002Q^2 - 5Q + 70,000 \end{aligned}$$

Profit function can be written as,

$$\begin{aligned} \text{Total Profit} &= (P - C)Q \\ &= \{ (70,000 - 3Q) - (0.0002Q^2 - 5Q + 70,000) \} Q \\ &= \{ 70,000 - 3Q - 0.0002Q^2 + 5Q - 70,000 \} Q \\ &= 2Q^2 - 0.0002Q^3 \end{aligned}$$

Differentiating above equation with respect to Q, we get

$$\frac{d(\text{Profit})}{dQ} = 4Q - 0.0006Q^2$$

$$\frac{d^2(\text{Profit})}{dQ^2} = 4 - 0.0012Q$$

Now for maxima or minima, we have

$$4Q - 0.0006Q^2 = 0$$

$$\Rightarrow Q(4 - 0.0006Q) = 0$$

$$\Rightarrow Q = 0 \text{ or } (4 - 0.0006Q) = 0$$

$$\Rightarrow Q = 0 \text{ or } Q = \frac{4}{0.0006}$$

i.e. $Q = 0$ or $Q = 6,666.667 \approx 6,667$ units.

At $Q = 0$, the value of the second derivative is $= 4 - 0.0012 \times 0 = 4$,

Therefore the value of the profit function is **minimum** at this point.

At $Q = 6,667$ the value of the second derivative $= 4 - 0.0012 \times 6667 = -4.0004$

Therefore the profit is **maximum** at $Q = 6667$ number of units.

$$\begin{aligned} \text{The maximum profit that can be obtained} &= 2Q^2 - 0.0002Q^3 \\ &= 2 \times 6667^2 - 0.0002 \times 6667^3 \\ &= \text{Rs } 2,96,29,629.41 \\ &\approx \text{Rs } 2.96 \text{ Crores.} \end{aligned}$$

67. Answer : (b)

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Reason : **Median:**

$$\text{Position of median term} = \frac{n+1}{2} = \frac{200+1}{2} = 100.5$$

Class	Frequency (f)	Cumulative frequency (cf)
4-5	31	31
5-6	82	113
6-7	54	167
7-8	21	188
8-9	12	200

∴ Median class is 5-6.

$$\text{Median} = \left[\frac{(N+1)/2 - (F+1)}{f_m} \right] W + L_m$$

$$L_m = 5; \quad N = 200 \quad F = 31, \quad W = 1, \quad f_m = 82.$$

$$\text{Median} = \left[\frac{(201/2) - (31+1)}{82} \right] \times 1 + 5 = 5.84 \text{ (approx.)}$$

68. Answer : (d)

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$$\text{Reason : a. } P(3 \text{ aces}) = \frac{{}^4C_3 \times {}^{48}C_1}{{}^{52}C_4} = \frac{4 \times 48}{270725} = \frac{192}{270725}$$

(Note that out of 52 cards there are 4 aces and $52 - 4 = 48$ non-aces. Since, three cards will be aces one will be a non-ace card.)

b. P (King, Queen, Jack and Ace in any order)

$$= \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_4} = \frac{256}{270725}$$

$$\text{c. } P(2 \text{ kings and 2 queens}) = \frac{{}^4C_2 \times {}^4C_2}{{}^{52}C_4} = \frac{36}{270725}$$

$$\text{d. } P(3 \text{ aces and 1 king}) = \frac{{}^4C_3 \times {}^4C_1}{{}^{52}C_4} = \frac{4 \times 4}{270725} = \frac{16}{270725}$$

e. There are 48 non-aces

$$\therefore P(\text{No ace is obtained}) = \frac{{}^{48}C_4}{{}^{52}C_4} = \frac{194580}{270725}$$

In all other events there will be at least one ace.

$$\therefore P(\text{At least one ace}) = 1 - P(\text{No ace is obtained})$$

$$= 1 - \frac{194580}{270725} = \frac{76145}{270725}$$

69. Answer : (c)

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Reason : Let the following notations be used:

S : Selection of a stock from the entire group of 30 stocks and 20 bonds.

B : Selection of a bond from the entire group of 30 stocks and 20 bonds.

I : Decision for investment will be made.

$$\text{a. } P(S) = \frac{30}{30+20} = \frac{30}{50} = 0.60$$

$$P(B) = \frac{20}{30+20} = \frac{20}{50} = 0.40$$

$$P(I) = 32\% = 0.32 \text{ (given)}$$

$$P(S \text{ and } I) = 24\% = 0.24 \text{ (given)}$$

$$P(I) = P(S \text{ and } I) + P(B \text{ and } I)$$

$$\text{or } 0.32 = 0.24 + P(B \text{ and } I)$$

$$\text{or } P(B \text{ and } I) = 0.32 - 0.24 = 0.08$$

∴ Probability that a bond is selected by Gopal and a decision to invest in it will be made = $0.08 = 8\%$.

b. $P(B \text{ and } I) = P(B) \cdot P(I/B)$

$$\text{or } 0.08 = (0.40) P(I/B)$$

$$\text{or } P(I/B) = \frac{0.08}{0.40} = 0.20$$

$P(I/B) + P(I'/B) = 1$ where, I' = No decision for investment will be made

$$\therefore P(I'/B) = 1 - P(I/B) = 1 - 0.20 = 0.80$$

$$\therefore P(B \text{ and } I') = P(B) \cdot P(I'/B) = (0.40)(0.80) = 0.32 \text{ i.e. } 32\%$$

∴ Probability that a bond is selected by Gopal and no decision to invest in it will be made = 32% .

c. $P(S) = P(S \text{ and } I) + P(S \text{ and } I')$

$$\text{or } P(S \text{ and } I') = P(S) - P(S \text{ and } I) = 0.60 - 0.24 = 0.36 \text{ i.e. } 36\%$$

∴ Probability that a stock is selected by Gopal and no decision to invest in it will be made = 36% .

d. $P(I \text{ and } B) = P(I) \cdot P(B/I) = P(B \text{ and } I)$

$$\therefore P(B/I) = \frac{P(B \text{ and } I)}{P(I)} = \frac{0.08}{0.32} = 0.25 \text{ i.e. } 25\%$$

∴ Probability that Gopal selected a bond if a decision for investment is made = 25%

e. $P(I \text{ and } S) = P(I) \cdot P(S/I) = P(S \text{ and } I)$

$$\therefore P(S/I) = \frac{P(S \text{ and } I)}{P(I)} = \frac{0.24}{0.32} = 0.75 \text{ i.e. } 75\%$$

∴ Probability that Gopal selected a stock if a decision for investment is made = 75% .

70. Answer : (d)

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Reason : The given series, $4\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 + \dots\infty$
is of the form, $a + ar + ar^2 + ar^3 + ar^4 + \dots$

The sum of a series of this form,

$$S = \frac{a}{1-r}$$

$$a = 4\left(\frac{1}{2}\right); r = \frac{1}{2}$$

$$\therefore S = \frac{4\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 2 \times 2 = 4$$

71. Answer : (e)

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Reason : $2x - y + 3z = 7$ (A)

$x + y - 2z = 6$ (B)

$x + 2y - 4z = 7$ (C)

Adding equations (A) and (B) _____

$$\begin{array}{rcl}
 2x - y + 3z & = & 7 \\
 \underline{x + y - 2z} & = & \underline{6} \\
 3x & + & z = 13 \quad \text{..... (D)}
 \end{array}$$

Multiplying equation (A) with 2 and adding it to equation (C) _____

$$\begin{array}{rcl}
 4x - 2y + 6z & = & 14 \\
 \underline{x + 2y - 4z} & = & \underline{7} \\
 5x & + & 2z = 21 \quad \text{..... (E)}
 \end{array}$$

Multiplying equation (D) with 2 and subtracting equation (E) from it _____

$$\begin{array}{rcl}
 6x + 2z & = & 26 \\
 \underline{5x + 2z = 21} & & \\
 x & = & 5
 \end{array}$$

Putting the value of x in equation (D) _____

$$\begin{array}{rcl}
 3(5) + z & = & 13 \\
 \text{or } 15 + z & = & 13 \\
 \text{or } z & = & 13 - 15 = -2
 \end{array}$$

Putting the values of x and z in equation (B)

$$\begin{array}{rcl}
 5 + y - 2(-2) & = & 6 \\
 \text{or } 5 + y + 4 & = & 6 \\
 \text{or } y + 9 & = & 6 \\
 \text{or } y = 6 - 9 & = & -3
 \end{array}$$

The values of the variables are

$$x = 5; y = -3 \text{ and } z = -2.$$

72. Answer : (c)

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Reason : Let E_1 be the event that a ball is drawn from first bag, E_2 the event that a ball is drawn from the second bag and E the event a black ball is chosen.

Ball is chosen from the first bag if the die shows 1 or 3. The probability of this event is $P(E_1)$

$$\frac{2}{6} = \frac{1}{3}.$$

If other faces turn up, the ball is chosen from the other bag, the probability of this event is

$$P(E_2) = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$\text{Now } P(E/E_1) = \text{Probability of choosing black ball from the first bag} = \frac{{}^3C_1}{{}^7C_1} = \frac{3}{7}$$

$$\text{Now } P(E/E_2) = \text{Probability of choosing black ball from the second bag} = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

Hence the probability of choosing black ball = $P(E) = P(E_1) P(E/E_1) + P(E_2) P(E/E_2) =$

$$\frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{11}{21}.$$

73. Answer : (c)

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Reason : If $f(x) = \ln g(x)$, then $f'(x) = \frac{g'(x)}{g(x)}$

$$\frac{d}{dx} \ln(x^4 + 4) = \frac{4x^3}{x^4 + 4}$$

$$\therefore \int_1^2 \frac{4x^3}{x^4 + 4} dx = \left[\ln(x^4 + 4) \right]_1^2 = \left[\ln(2^4 + 4) - \ln(1^4 + 4) \right]$$

$$= \left[\ln 20 - \ln 5 \right] = \ln \left(\frac{20}{5} \right) = \ln 4$$

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